

DAN:

Sea x el angulo entre dos vectores \mathbf{u} y \mathbf{v} en R^2 o R^3 . Entonces

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \cdot \|\mathbf{v}\| \cdot \cos x$$

$$\cos x = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \cdot \|\mathbf{v}\|}$$

PIDEN:

Demostrar que:

$$\bullet \quad \|\mathbf{v} - \mathbf{u}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 \cdot \cos x$$

EJECUCION:

$$\begin{aligned} \|\mathbf{v} - \mathbf{u}\|^2 &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 \cdot \cos x \\ 2\|\mathbf{u}\| \|\mathbf{v}\| \cdot \cos x &= \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2 - \|\mathbf{v} - \mathbf{u}\|^2 \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} - (\mathbf{v} - \mathbf{u}) \cdot (\mathbf{v} - \mathbf{u}) \\ &= \mathbf{u} \cdot \mathbf{u} + \mathbf{v} \cdot \mathbf{v} - \mathbf{v} \cdot \mathbf{v} + \mathbf{v} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} - \mathbf{u} \cdot \mathbf{u} \\ &= 2\mathbf{u} \cdot \mathbf{v} \\ \text{luego } \mathbf{u} \cdot \mathbf{v} &= \|\mathbf{u}\| \cdot \|\mathbf{v}\| \cos x \end{aligned}$$

BIBLIOGRAFIA:

Algebra lineal (octava edicion) - Bernard Kolman . David R. Hill